Number Sense: Rethinking Arithmetic Instruction for Students with Mathematical Disabilities

By: Russell Gersten and David J. Chard (2001)

Abstract

We describe the concept of number sense, an analog as important to mathematics learning as phonemic awareness has been to the reading research field. Understanding the concept of number sense and relevant research from cognitive science can help the research community pull together fragmented pieces of earlier knowledge to yield a much richer, more subtle, and more effective means of improving instructional practice.

More than three decades have passed since Kirk and Bateman (1962) proposed that auditory processing was one of the psycholinguistic process deficits underlying specific learning disabilities. Although subsequent psychometric studies identified the flaws in their conceptualization, our current understanding of the importance of phonological processing and its contribution to reading development suggests that Kirk and Bateman were at least partly accurate in their analysis. In fact, the most notable advances in the learning disabilities field since the late 1970's have been in reading disabilities, a subtype of learning disabilities. More specifically, important advances have been realized in the prevention (e.g., Ball & Blachman, 1991; Byrne & Fielding-Barnsley, 1993; Felton & Pepper, 1995; Felton & Wood, 1992) and remediation (e.g., Torgesen, Wagner, Rashotte, Alexander, & Conway, 1997) of reading difficulties. These advances are primarily the result of the growing knowledge base on phonemic awareness and its importance to the development of strong reading skills.

We believe that there may be an analog as important to mathematics learning as phonemic awareness has been to the development of reading. Our goal in this article is to introduce this analog. To accomplish our goal, we briefly review the concept of phonemic awareness and its crucial role in helping students with learning disabilities to learn to read. Then, we demonstrate how this concept helps the research community pull together fragmented pieces of earlier knowledge and yield a much richer, more subtle, and more effective means for improving instructional practice than earlier conceptions (e.g., Kirk & Bateman, 1962). Furthermore, we describe what we view as important findings concerning children's acquisition of mathematical concepts, by developing the idea of number sense. We demonstrate how the number sense concept can inform and significantly enhance the quality of mathematics interventions for students with learning disabilities, just as the concept of phonemic awareness has informed the field of reading. The number sense concept acts as a lens to reveal reasons for relative successes and failures of past attempts at innovations. In particular, we review the research of Hasselbring, Goin, and Bransford (1988) and Pellegrino and Goldman (1987) from a contemporary perspective. We conclude with a model of understanding specific learning disabilities adapted from Kolligian and Sternberg (1987), and Geary (1990, 1993, 1994). We demonstrate how this can he a useful framework for conceptualizing interventions.

Our model is linked to issues generated by the mathematics reform movement, as reflected in the National Council of Teachers of

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Mathematics (NCTM; 1989) and the reactions to these guidelines (Carnine, 1997; Hofmeister, 1993; Mercer, Jordan, & Millet, 1994; Rivera, 1997). Our model indicates how the number sense concept provides a sensible middle ground in what is becoming an increasingly heated controversy about how to teach mathematics.

In our approach, we rely on Cobb's (1995) conceptualization of constructivism as a joint approach. In this conceptualization, mathematical learning occurs as students (a) learn the conventions, language, and logic of a discipline such as mathematics from adults with expertise: and (b) actively construct meaning out of mathematical problems (i.e., try a variety of strategies to solve a problem). We believe that cognitive insights can, and should have a profound impact on how math is taught to special education students and can help radically reform the mundane drill and practice typical of special education mathematics instruction.

In this article, we draw analogies between phonological awareness and number sense. We also draw analogies between earlier research on ways to remediate mathematical disabilities and earlier research on reading disabilities. The goal here is to provide a brief overview of phonological awareness concepts and number sense before introducing the concept of number sense, rather than to attempt to provide a comprehensive review of either topic.

Overview of phonological awareness

In the reading field in the 1980s, based on insights gained from cognitive psychologists such as Perfetti (1986), the consensus was largely that fluent, virtually automatic decoding was essential for comprehension. As a consequence, sustained efforts were critical to transform students with learning disabilities into fluent readers. Typically, there were two components: (a) sustained systematic work on phonics as a means to "break the code" and build proficiency, and (b) the use of repeated readings to build fluency (O'Shea, Sindelar, & O'Shea, 1987; Samuels, 1986). Attempts to address underlying auditory or visual processing deficits, such as those historically associated with learning disabilities (Kirk & Bateman, 1962), were consistently viewed as ineffective (Hallahan & Kauffman, 1977; Kavale & Mattson, 1983; Ysseldyke, 1973). There was, perhaps, an underlying sense that merely teaching phonics was insufficient for helping students with disabilities to learn to read. Yet, there were some findings that could not be explained by this view. Kavale (1984) for example, found that correlations between auditory skills and reading were consistently positive and consistently replicated. Similarly, Gersten and Carnine (1985) noted that, in particular, the skill of auditory blending was highly correlated with reading measures.

Adams (1990) synthesized the body of research on early reading and popularized the concept of phonemic awareness. Adams (1990) noted that measures of phonemic awareness were the best predictors of early reading performance, that is, better than IQ, readiness test scores, or socioeconomic level.

Phonemic awareness is the insight that words are composed of sounds. Although this insight is not necessary to speak or understand spoken language, it is essential for reading acquisition (Wagner & Torgesen, 1987). Phonemic awareness is not always easy for children to obtain. As Williams (1995) noted,

> Phonemes are abstract units, and when one pronounces a word one does not produce a series of discrete phonemes; rather phonemes are folded into one another and are pronounced as a blend: Although most young children have no difficulty segmenting words into syllables, many find it very difficult to segment at the phoneme level. (p. 185)

A host of studies has linked phonemic awareness to the development of reading skills (Stanovich, Cunningham, & Cramner. 1984; Vellutino & Scanlon, 1987; Williams, 1984). Another series of studies has demonstrated that instruction in phonemic skills leads to enhanced reading performance (Cunningham, 1990; Lie, 1991; O'Connor, Notari-Syverson. & Vadasy, 1996; Torgesen, Morgan, & Davis, 1992). However, the mass of intervention studies indicates that children with average development have benefited more from phonemic awareness instruction than children at risk for learning disabilities or children with cognitive impairments (Smith, Simmons, & Kameenui, 1998). The critical insight into the connection between sound and print may be "hard won" (Blachman, 1994; Wagner, Torgensen, & Rashotte, 1994) for children who have difficulty responding to phonemic awareness and reading instruction. Nevertheless, an emerging base of empirical evidence suggests that more intense interventions at the kindergarten level, as well as longitudinal interventions may provide necessary benefit for those children.

Current thinking about special education reading instruction, both remedial and preventive, now invariably notes the importance of explicit instruction in phonemic awareness skills. For example, we have learned from research that explicit training in sound blending is useful to students. However, students are helped even more if they are provided with instruction not only in how to blend phonemes together, but also in how to "pull apart" or segment words into phonemes (Smith et al:, 1998).

The concept of phonemic awareness has helped connect the pieces of the puzzle of reading acquisition. Phonemic awareness provides greater precision and helps to inform instruction in a way that earlier concepts of phonics instruction, which rarely included instruction in either blending or segmentation, did not. It is extremely important to note that the findings regarding phonemic awareness and its importance to reading acquisition do not diminish the importance of automaticity of decoding for strong reading comprehension. However, the findings lead to a decrease in the "brute force"' drill emphasis of many early phonics programs, because we are clearer and can be more strategic about what we need to teach in beginning reading.

Just as our understanding of phonemic awareness has revolutionized the teaching of beginning reading, the influence of number sense on early math development and more complex mathematical thinking carries implications for instruction.

What is number sense?

Those who plan mathematics instruction for young children fail to take fully into account that, along with increased competence and fluency with basic addition and subtraction facts, children also develop- or fail to develop- a number sense. Number sense is an emerging construct (Berch, 1998) that refers to a child's fluidity and flexibility with numbers, the sense of what numbers mean and an ability to perform mental mathematics and to look at the world and make comparisons. Struggling to define number sense, Case (1998) stated the following:

Number sense is difficult to define but easy to recognize. Students with good number sense can move seamlessly between the real world of quantities and the mathematical world of numbers and numerical expressions. They can invent their own procedures for conducting numerical operations. They can represent the same number in multiple ways depending on the context and purpose of this representation. They can recognize benchmark numbers and number patterns: especially ones that derive from the deep structure of the number system. They have a good sense of numerical magnitude and can recognize gross numerical errors that is, errors that are off by an order of magnitude. Finally, they can think or talk in a sensible way about the general properties of a numerical problem or expression-- without doing any precise computation. (p. 1)

Most children acquire this conceptual structure informally through interactions with parents and siblings before they enter kindergarten. Other children who have not acquired it require formal instruction to do so (Bruer, 1997). For example, one child may enter school knowing that 8 is 3 bigger than 5, whereas a peer with less well-developed number sense may know only that 8 is bigger than 5. Other children may have very well-developed number sense and may have a strategy for figuring out how much bigger 8 is than 5 using fingers or blocks.

This number sense not only leads to automatic use of math information, but also is a key ingredient in the ability to solve basic arithmetic computations. Knowing that 15 is much further away from 8 than 11 requires an awareness of 8 + 7 and 8 + 3. However, more than 100 basic addition facts must be memorized to automaticity before students can experiment with this type of interesting problem.

Griffin, Case, and Siegler (1994) suggested that number sense is often informally acquired prior to formal school and is a necessary condition for learning formal arithmetic in the early elementary grades. Bruer (1997) detailed bow research since the late 1970's has provided evidence of a preverbal component to number sense. By age 3 or 4 years, most children can compare two small numbers for size and determine which is larger and which is smaller.

Number sense is facilitated by environmental circumstances. As with phonemic awareness, the environmental conditions that promote number sense are, to some extent, mediated by informal teaching by parents, siblings, and other adults. For example, Griffin et al. (1994) found that entering kindergartners differed on questions such as "which number is bigger, 5 or 4?" even when they controlled for student abilities in counting and working simple addition problems in the context of visual materials. High socioeconomic status (SES) children answered the question correctly 96% of the time, compared with low SES children who answered correctly only 18% of the time. Adams (1990) carefully documented how home-based activities associated with the development of phonemic awareness typically are common everyday activities in the homes of middle-class families and much less likely to be an everyday part of low SES homes. Griffin and Case (cited in Phillips & Crowell, 1994) documented the same phenomenon related to number sense. On average, in well-educated middle-class homes, there is a good deal of informal instruction about numbers and concepts related to numbers such as two more or double and, on average, significantly less of this type of instruction in low SES homes.

The notion of number sense has enjoyed intuitive, almost romantic, support previously. It is even common to hear educators comment that some students are "just good with numbers" or generalize about the mathematics prowess of certain groups of students.

We contend that number sense is more than a common parlance notion. There is increasing empirical support for its relationship to underlying deficits in learning disabilities (Geary, 1993; McCloskey & Macaruso, 1995) and some support that instruction including number sense activities leads to significant reductions in failure in early mathematics (Griffin et al., 1994). Moreover, we submit that simultaneously integrating number sense activities with increased number fact automaticity rather than teaching these skills sequentially- advocated by earlier special education mathematics researchers such as Pellegrino and Goldman (1987)- appears to be important for both reduction of difficulties in math for the general population and for instruction of students with learning disabilities. It is also likely that some students who are drilled on number facts and then taught various algorithms for computations may never develop much number sense, just as some special education students, despite some phonics instruction and work on repeated readings/fluency and accuracy, fail to develop good phonemic awareness or any sense of the purpose or pleasure of reading.

The number sense construct that is the focus of our attention here seems analogous to phonemic awareness in several ways. We want to stress that, like all analogies, this one is far from perfect. There are numerous differences between the development of an understanding of and proficiency in mathematics and the development of the ability to read with understanding. For example, beginning reading clearly involves a heavy auditory component, whereas number sense is much less dependent on auditory processing. Nonetheless, we believe this analogy can be helpful in conceptualizing directions for improvement of math instruction for students with learning disabilities. In particular, we believe that if beginning math instruction were focused in part on building number sense, many students with learning disabilities would benefit.

Similar to phonemic awareness instruction, we are not sure of the best approach to use to teach number sense. Although some programs seem to have the potential to reach number sense through strategic practice on math facts (Phillip & Crowell, 1994; Tarver & Jung, 1995: Willis & Checkley, 1996), number sense is likely to be necessary but not sufficient for developing problem-solving skills. After all, we understand that phonemic awareness is necessary for fluent reading and ultimately for reading comprehension, while also recognizing that many other components are necessary for comprehension (Lyon, 1994).

Earlier special education researchers attempted to increase automaticity with math facts by systematic drill and practice because of the correlation between automaticity and mathematics competence (Pellegrino & Goldman, 1987). But this "brute force" approach made mathematics unpleasant, perhaps even punitive, for many. In addition, it had the effect of disassociating mathematics facts from mathematical reasoning, just as some earlier approaches toward phonics instruction separated practice with sounds from the actual blending of sounds into words or needlessly separated reading instruction from the experience of reading.

Early special education mathematics research: Building automaticity with basic facts

In this section, we refer to the research conducted in the 1980s by Hasselbring et al. (1988) and Pellegrino and Goldman (1987) on effective means to teach mathematics to students with learning disabilities using technology. In our view, this research remains relevant, but as with earlier reading research (Carnine, 1976, 1977; Samuels, 1979), we can now revisit it from a more contemporary perspective. With recent investigations in cognitive science, we are now in a better position to see how various pieces fit together and design more sophisticated interventions that are more likely than earlier approaches to engage students with learning disabilities.

Conceptual basis of automaticity research in mathematics

Research on automaticity in mathematics was based on a model of mathematics quite analogous to earlier models of reading and reading disability. Essentially, the prevailing hypothesis was that students with learning disabilities tend not to recall basic math facts automatically (Fleischner, Garnett, & Shepard, 1982; Hasselbring et al., 1988). Researchers cited descriptive research demonstrating that, even by age 7 years, students with learning disabilities recalled significantly fewer facts from memory than students without disabilities, and that each year, the gap widens. For example, by age 12 years, with students with average ability recall, on average, triple the number of basic math facts that their peers with learning disabilities recall (Hasselbring et al.. 1988). It is important to note that differences in accuracy between the groups are fairly trivial. By this age, most students with learning disabilities are able to successfully perform simple addition and subtraction problems, but they tend to rely on counting fingers of stick figures. By contrast, the average student increasingly provides a response to a problem such as 9 + 8 automatically.

Researchers explored the devastating effects of the lack of automaticity in several ways. Essentially, they argued that the human mind has a limited capacity to process information, and if too much energy goes into figuring out what 9 plus 8 equals, little is left over to understand the concepts underlying multi-digit subtraction, long division, or complex multiplication. We also argue that, as researchers increasingly attempt to teach math following NCTM: guidelines, students' comprehension of what the teacher is discussing is likely to be limited because the teacher assumes such automaticity as a basis for explanations. For example, Willis and Checkley (1996) described a teacher who derives math problems from real-world events, such as the floods last winter, and from themes her pupils study, such as transportation. Such an approach is designed to help students connect their knowledge of mathematics to important and personally relevant problems. However, a student with learning disabilities in this class, who way not have automatic access to facts such as 32 is 8 more than 24 and that 32 is double 16, may not comprehend much of the class discussion and may fail to increase understanding of the concepts taught and discussed. In fact, a study by Woodward and Baxter (1997a) found that students with disabilities in mathematics tended to make significantly less growth in discussion-oriented classrooms than students with disabilities taught with more traditional methods.

This focus on the devastating effects of weak automaticity on the ability to solve problems and understand mathematical concepts is a direct parallel to the reading research of the early 1980s, which demonstrated that students who are slow or plodding decoders tend to be poor comprehenders.

Instructional research to increase automaticity

Instructional research to increase automaticity in the areas of basic arithmetic facts and operations was conducted in the 1980s. Pellegrino and Goldman (1987) stated that

the general concept of automaticity….is that, with extended practice, specific skills can reach a level of proficiency where skill execution is rapid and accurate with little or no conscious monitoring. ... Processing moves from a state in which demands are made on a limited attentional resource pool to a state where fewer demands are made on those resources. ...It is assumed that the freed attentional resources can be allocated to other tasks or processes, including higher-level executive or control function. ...Automatic processes are relatively effortless, make few attentional demands, and operate concurrently with other task components. (pp. 145-146)

Pellegrino and Goldman (1987) concluded that the focus of mathematics remediation for students with learning disabilities should involve extended practice on math facts for which the student still relied on counting procedures. They argued that extended practice would lead to "development of a degree of automaticity that affords them the attentional and resource opportunities to engage in metacognitive activities… being able to allocate more attention to higher-order aspects of the task or to restructuring of performance patterns" (Pellegrino & Goldman, 1987, p. 146). Using conceptions of cognitive processes prevalent at the time, they argued that basic math facts must become declarative knowledge so that the students can devote energies to higher-order thinking. Most students with learning disabilities possessed procedural knowledge of these basic math facts (i.e., they could correctly calculate the sum of 6 + 8), but they need to store these facts in memory in a manner that allows for retrieval "quickly, effortlessly and without error" (Hasselbring et at., 1988, p. 2).

The next research phase involved a series of studies (Hasselbring et al., 1988) designed to significantly enhance the automaticity of addition facts for students with learning disabilities using computer-assisted instruction. A sophisticated instructional program was established in which students were provided with individualized daily practice for approximately 10 minutes per day. The drill-and-practice software included numerous features of sophisticated instructional design: (a) immediate feedback on incorrect responses because lack of immediate feedback on incorrect responses can be detrimental to cognitive growth (Pellegrino & Goldman, 1987; Siegler & Shrager, 1984); and (b) large amounts of practice, which are necessary to ensure automatic retrieval.

The program used an interspersal of target facts with already automatic facts to maximize practice time. Practice continued until the student consistently used retrieval as opposed to counting on his or her fingers. Controlled response times were also used to force students to not rely on counting. Hasselbring et al. (1988) concluded that this was probably the critical principle in the instructional design. The researchers also determined that use of computer-aided instruction was effective in increasing automaticity for most, but not all, of the students with learning disabilities. Hasselbring et al. (1988) noted that if students relied solely on counting fingers and did not attempt to recall addition facts from memory (i.e., did not have any established declarative networks), then the extended practice was not helpful. If, however, they did recall facts from memory, albeit slowly, even 4 weeks of extended drill and practice often led to automatic retrieval of those facts. Dramatic increases in automatic responding were found in both sets of studies. For example, in one large-scale study, the number of facts automatically retrieved increased from 27.5 to 45 after 50 sessions. This concerted effort to "force" students to rely less on counting (procedural knowledge) and more on declarative knowledge (retrieval from memory) appeared to be quite successful.

This research appeared to restructure (Cheng, 1985) how many students with learning disabilities approached the task. The ultimate goal was to "free up" mental resources for performance. This approach had direct and indirect effects on the field of special education math practice (Miller & Mercer, 1997; Montague, 1997; Parmar & Cawley, 1997). Various mental math activities were implemented in special education settings and an increased awareness of the importance of automatic retrieval of math facts slowly spread in the special education math community.

This type of drill and practice did not take into consideration the fact that "math facts" are always, at some point in time, problems to solve (Siegler, 1988). They also failed to take into account that (a) given the proper classroom environment (e.g., contextualizing instruction), seemingly mundane computation exercises may be fascinating to children (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Lampert, 1986); or (b) students team specific strategies as they automatize math facts (Baroody, 1992; Siegler, 1988; Silbert, Carnine, & Stein, 1989). The following section examines research on the development of mathematical thinking and considers instructional implications.

Adjusting instruction to address individual differences in mathematical development

Studies from both cognitive development (Shrager & Siegler, in press; Siegler. 1988; Griffin et al., 1994) and instructional research (Case, 1998; Griffin, 1998) are beginning to provide a deeper understanding of mathematical strategies that children often naturally develop for handling arithmetic problems. These studies are also providing some insight into means for developing these strategies in students with mathematical disabilities through well-designed instruction. In an effort to examine the consistencies in young children's strategy choices on addition and subtraction problems and word identification, Siegler (1988) performed two experiments using cluster analysis to distinguish classes of children based on the accuracy of their performance and on their strategy use. Three groups were identified: those who performed well, those who performed poorly, and those who Siegler called perfectionists. Siegler's (1988) findings illustrated individual differences not only in children's knowledge, but also in their impulsivity in applying backup strategies to solve addition and subtraction fact problems. For example, a key strategy that most children learn is the "min" strategy-that it is more efficient to start with the larger number than the smaller one when trying to find the answer to either 3 + 8 or 8 + 3 if using one's fingers, manipulatives, or stick figures. Acquisition of this mini-strategy is an essential predictor of success in early mathematics. Children often learn or deduce this strategy. However, Siegler (1988) found that some children do not acquire the strategy readily. These less successful children seem to represent students with learning disabilities and students who are at risk for school failure.

In Siegler's (1988) words, the educational implication is "that it might be useful to teach children, particularly not-so-good students, to more accurately execute backup strategies" (p. 850). Although this implication is not a revelation to many educators, it is in contrast to the approach typically used to teach arithmetic. In fact, Siegler's (1988) findings suggested that arithmetic facts represent strategy-based problems for low performers (both children with learning disabilities and those at risk for school failure). As a result, we assume that these low performers will need systematic instruction in strategy use that others may not. Siegler (1988) described the intuitive benefits of this differential instruction as follows: "Teaching children to execute backup strategies more accurately affords them more opportunities to learn the correct answer, reduces the likelihood of associating incorrect answers, produced by faulty execution of backup strategies, with the problem" (p. 850).

Other research offers valuable additional information to help guide instruction. In particular, Shrager and Siegler (1998) indicated that generalization of strategy use proceeds extremely slowly in young children. Adults often underestimate the time it takes for a child to consistently use a newly learned mathematical strategy.

Shrager and Siegler (1998) also found that, at least for very basic arithmetic strategies, "generalization increases greatly with presentation of 'challenge problems.' " (p. 7) These are problems that are very difficult to solve without use of the strategy and fairly easy to solve with the strategy. This finding, too, would seem to have important instructional implications for special education.

Methods for teaching strategies have been developed by Case (1998) and Griffin (1998). In their comparative research on arithmetic knowledge of low- and middle-income kindergartners, Case et al. (cited in Phillips & Crowell, 1994) found that when children who are mathematically naive enter school and are shown two groups of objects, they are often able to identify the "bigger group" and to stipulate that the bigger group has "a lot more." However, they are unlikely to be able to specify how much bigger one group is than the other. For example, Case et al. noted that a substantial number of the low-income children in their study were unable to tell which of two single-digit numbers was larger or which of two single-digit numbers was closer to 5. They noted, however, that often this lack of knowledge can be traced to a lack of explicit instruction in the home. Based on home observations, they inferred that these learning deficits reflected a lack of experiences with adults or siblings that would facilitate the association of quantity and numbers and would lead to the development of an abstract numerical understanding (i.e., basic number sense). This mental number line appears to be the critical "big idea" necessary for solving addition and subtraction problems common in first grade (Phillips & Crowell, 1994; Tarver & Jung, 1995).

Griffin (1998) proceeded to demonstrate that schools could provide guided instruction that builds number sense in kindergartners who enter with deficits in the area of abstract mathematical reasoning. The goal of this instruction is in large part, for students to develop and elaborate an integrated schema that centers on a mental number line, allowing students to solve a variety of addition and subtraction problems. In contrast to other approaches, which call for teaching in multiple modalities, with the Griffin (1998) and Case (1998) approach, only three representational systems are used. The first is conventional mathematical symbols (digits, addition, subtraction, and equal signs). The second and third are intended to foster the sense of the number line. They are a thermometer which shows the number line in a clear vertical direction, so that students see that, with this representation, bigger corresponds to higher and smaller corresponds to lower. The vertical representation is an excellent means for initially building number sense because correspondences between bigger and up or addition and "'going up" are clear and unequivocal. However, the researchers were concerned that students would stipulate on this one-to-one correspondence so students also use a representational system that looks a bit like the Candyland game. Here the correspondences are subtler. By using only three modalities, the students can easily trace consistencies across all three, whereas when too many representational systems are used, some students, especially those with disabilities, may not notice the similarities.

Other cornerstones of the approach used by Griffin (1998) and Case (1998) are the frequent opportunities for students to verbalize their understandings and rationales for the strategies they use to solve problems and the use of extensive practice in solving problems using the three modalities. The former is based on the belief, with extensive support in the reading comprehension research literature (Gersten et al.. 1998; Mastropieri, Scruggs, Bakken, & Whedon, 1996; Pearson & Dole, 1987) and the expressive writing literature (Englert, Raphael, Anderson, Anthony, & Stevens, 1991; Graham & Harris, 1997; Wong, 1994), that encouraging students to verbalize their current understandings and providing feedback to the student increases learning. The latter is modeled, in large part, on Japanese methods of teaching mathematics.

These preliminary findings are extremely promising and the cornerstones of this approach appear to have relevance for special education mathematics instruction. Also relevant are the insights gained by Siegler and Stern (1998) in their studies of how students develop efficient strategies for solving problems. They observed, for example, how children learned to solve what they called the inversion problem (A + B - B =A). In other words, they observed how students inferred this principle after being provided a range of problems, such as "What is 6 plus 3 minus 3? What is 4 plus 8 minus 4?" Example selection appeared to be a key factor that enhanced learning of this principle. The authors noted that both computational speed and accuracy and conceptual understanding influenced development of the strategy for efficiently solving the problem. They argued that math instruction needs to take all these facets into account.

Instructional implications

Just as specialized, highly specific instructional practices in early literacy activities can have significant benefits for students when they are implemented early, similar practices may be beneficial for children who are lacking early numeracy experiences or students with learning disabilities. To address phonemic awareness difficulties, curriculum materials and instructional approaches developed by Blachman (1994), O'Connor et at. (1996), and Torgesen et al. (1992), among others, have proved to be successful in boosting the phonemic awareness and subsequent reading ability of many students, Similarly, there is evidence that curricula such as Number Worlds have the potential to help students develop number sense while also building computational and problem-solving skills (Griffin, 1998).

Early interventions focusing on prenumeracy skills attempt to expose children to experiences lacking in their home or in preschool. For example, parents can help children develop early number sense by asking them to ascend and count four steps and then count and descend two steps. Similarly, parents might ask their children to set the table and count the correct number of place settings. This requires children to map the number of forks, knives, and spoons with the number of people eating the meal. Griffin et al. (cited in Phillips & Crowell, 1994) referred to such experiences as "mini-lessons" in mathematical concepts that parents provide to their children. These early prenumeracy experiences form an analogy (however imperfect) to emergent literacy experiences described by Teale and Sulzby (1986).

Adams (1990) noted that children develop an awareness that letters determine sounds in words. However, many children need explicit, consistent help in understanding the specifics of the system. In other words, they possess very crude levels of phonemic awareness and need help developing the sophisticated awareness necessary for fluent, nonstressful reading. Part of phonemic awareness is an awareness that words are composed of sounds; this awareness, coupled with an understanding that the alphabet represents sounds, helps children know that reading involves putting sounds together to create words. With Siegler's (1988) "min" strategy, a similar phenomenon occurs. There are isolated incidents when it does not matter whether a child can execute the strategy. Nevertheless, over the long haul, the strategy is essentially a sophisticated sense of proportion, concepts of equality, and how addition works.

It is important to note at this point that strategies such as the "min" strategy are not easy to teach. To recall quickly that 8 is bigger than 3, a child must have some factual automaticity. Also, a child needs a sense of numbers to assist in access or automaticity. Children need to master all three components: (a) problem-solving strategies, (b) verbal comprehension, and (c) automaticity with relevant facts. In a sense, our current knowledge base consists of our understanding that children differ in their sense of numbers, their representation of problems, and their application of strategies that integrate an the previous components to solve even basic arithmetic problems.

Examples of these differences have been identified in research. For example, Woodward and Howard (1994) reviewed computational performance among more than 100 middle school students with learning disabilities. Careful analysis of these tests revealed that more than half the students showed systematic error patterns, many of which revealed limited conceptual understanding of the algorithms and strategies taught to them. This finding is particularly troubling considering that many of these students were in the eighth grade and had been receiving mathematics instruction since first grade. One interpretation of the problems students with learning disabilities have with subtraction that requires regrouping is that this is the first math skill for which the child needs number sense to solve problems and, without such a sense, performance breaks down.

Links between number sense and specific cognitive problems experienced by students with learning disabilities

Research reported by neuropsychologists and cognitive psychologists converges on common core components of mathematical disabilities (Geary, 1993; Ginsburg, 1997; McCloskey & Macaruso, 1995). Geary (1993) characterized these major problems within the context of normal development of mathematical reasoning. The first is a high frequency of procedural errors. The second is difficulty in representation and retrieval of arithmetic facts. The third is an inability to symbolically or visually represent or code numerical information for storage in contrast to using words as the unit or form for storage in the brain (Geary. 1990; Geary & Brown, 1991). The first finding has been corroborated by Woodward and Howard (1994) and Cawley and Reines (1996). The second was confirmed by Pellegrino and Goldman (1987) and Hasselbring et al. (1988). We discuss these difficulties in the following section.

When presented with a basic arithmetic fact such as 8 + 5, children first typically attempt to retrieve the answer from memory. If incorrect, or unable to retrieve the fact, children apply the backup strategies that Siegler (1988) described (e.g., finger counting, "min" strategy, occasional use of manipulatives). This description of children's approaches to solving arithmetic problems corroborates Siegler's (1988) research discussed earlier.

Geary's (1993) review of cognitive and neuropsychological studies indicated that some problems that students with learning disabilities experience may be procedural. Procedural deficits include the inability to perform moderately complex addition, subtraction, multiplication, and division problems, as documented by Cawley and Reines (1996) and Woodward and Howard (1994).

The procedural deficits may disappear in later elementary grades if quality instruction is provided. In contrast, the deficits associated with coding numerical information in semantic forms (in contrast to visual forms) in the brain and retrieval of facts, in all likelihood, may represent a difference that persists. By way of a brief definition, cognitive psychologists use the term representation to refer to coding information in any form for storage in the brain.

The construct of number sense is likely to be related to semantic representation of information (e.g., mental number lines) and is critical to conceptual understanding, These representational problems lead to persistent problems in many areas of mathematics, especially those related to conceptual understanding and application of procedural knowledge to new problem-solving situations (Woodward & Baxler, 1997b).

A brief discussion of the link between math and reading- related deficits is appropriate. Many children with early reading difficulties exhibit auditory memory problems that show themselves through difficulty in performing phonological processing or phonemic awareness tasks (Adams, 1990; Olson, Gillis, Rack. DeFries, & Fulker, 1991; Olson, Wise, Conners, Rack, & Fulker, 1989). Many children who show phonologically based reading difficulties exhibit difficulties in arithmetic retrieval as well. This comorbidity of specific disabilities is well documented (Ashcraft. 1987, 1992: Light & DeFries, 1995). In fact, more than 60% of students with learning disabilities possess significant disabilities in mathematics (Light & DeFries, 1995).

Implications for practice

In many ways, mathematics has always been an afterthought in the learning disabilities field, which originated, after all, with a focus on reading and reading-related problems. In addition, the major source of referral and diagnosis for learning disabilities is problems in reading (Light & DeFries, 1995). The magnitude of research, curriculum development, and training in reading and language arts, as opposed to mathematics, would easily be of the order of 6 to 1 (Swanson, in press).

Pugach and Warger (1993) noted how some of the attempts of special educators "to foster cognitive-mediational or strategic learning strategies" have been disastrous because the teachers "isolate the strategies… from meaningful instruction and teach them as efficient prescriptions". (p. l34). Ginsburg (1997) suggested a partial solution by highlighting the need to focus on understanding (i.e., number sense). Instruction can target "figuring out the number facts" (p. 31) for students whose learning disabilities may lie in memory deficits that preclude their moving through the hierarchy of skills.

Too often, special education math instruction focuses on mastery of algorithms, repeated practice with limited opportunity for students to explain verbally their reasoning and receive feedback on their evolving knowledge of concepts and strategies. In other words, special education mathematics instruction continues to focus on computation rather than mathematical understanding.

By considering the analogy between phonemic awareness and number sense, it is hoped that we can focus attention on significant, necessary shifts in how mathematics is taught to young children, especially those with learning disabilities or those entering school with limited familiarity of arithmetic concepts.

At some point in time, even basic arithmetic facts are problems to be solved by naive learners, Therefore, mere drill and practice on basic math facts will be insufficient for developing students who are competent in mathematics. Even if students are not automatic with basic facts, they still should be engaged in activities that promote the development of number sense and mathematical reasoning.

It has been our goal in this article to review research that explicates the number sense construct, helps link it to recurring issues in special education math instruction, and provides some guidelines for further research in special education math instruction and the development of more effective instructional approaches to reduce the difficulties that students with disabilities often experience in mathematics. We believe that the first step in this process is an understanding of the importance of number sense, its relevance for building mathematical ability in students, and an understanding of limitations in current practice, much of which is not based on the concept of number sense.

We envision a wave of research and development on math instruction for students with disabilities that parallels research and development of instructional strategies and approaches related to the concept of phonemic awareness. Important first steps are the development and validation of measures that have reasonable predictive validity. To date, most of the assessments have been more clinical in nature. Predictive validity studies are likely to enhance our understanding of the construct, as well as provide important information as to which students are likely to be at risk for failure in the area of mathematics and require sensitive instruction in the early grades to reduce the risk of failure.

Equally important is increased research on effective beginning mathematics instruction using number sense as a construct in both curriculum development and assessment of effectiveness. There are some approaches. such as the work of Griffin (1998), that can serve as a starting point. The consistent use of no more than two representations of a mental number line seems to greatly assist students to develop fluency with basic arithmetic concepts and procedures (i.e., develop number sense).

Also germane are the important findings about the relationship between procedural and conceptual knowledge of Siegler and colleagues based on more than a dozen years of research on young children's development of arithmetic reasoning. Their findings suggest an intricate relationship between conceptual understanding and consistent use of efficient strategies for computation and problem solving. In reviewing this body of research, Rittle-Johnson and Alibali (in press) concluded:

Conceptual and procedural knowledge may develop interactively, with gains in one leading to gains in the other, which in turn trigger new gains in the first. Thus procedural knowledge could also influence conceptual understanding. ...Under some circumstances, children first learn a correct procedure and later develop an understanding of the concepts underlying it. (p. 6)

For example, most kindergartners "abstract out" the principle of commutativity from their experiences solving addition problems (Siegler, 1991). Student with learning disabilities, however, tend to have great difficulties abstracting principles from experiences (Geary, 1993; Swanson, 1987), and support is invariably necessary. Contemporary conceptions of the development of mathematical reasoning and computational proficiency in young children can serve as a basis for the development of much more effective approaches to mathematics instruction for students with disabilities than those that currently exist. The body of research provides a good direction for refinement and extension of current practice. This could be a potentially exciting era for what has become a neglected area in special education research.

Authors' note

This research was supported, in part, by a grant from the Spencer Foundation. The authors extend their thanks and appreciation to Scott Baker and Sylvia Smith for their thoughtful feedback on earlier versions of this manuscript. The authors also thank Kate Sullivan, Janet Otterstedt, Jon Gall, and Karie Hume for their editorial assistance.

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The Journal of Special Education (1999) 44:18-28 © 1999 by PRO-ED, Inc.

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