

Making Sense of Number Sense: Implications for Children With Mathematical Disabilities

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Abstract

Drawing on various approaches to the study of mathematics learning, Gersten, Jordan, and Flojo (in this issue) explore the implications of this research for identifying children at risk for developing mathematical disabilities. One of the key topics Gersten et al. consider in their review is that of "number sense." I expand on their preliminary effort by examining in detail the diverse set of components purported to be encompassed by this construct. My analysis reveals some major differences between the ways in which number sense is defined in the mathematical cognition literature and its definition in the literature in mathematics education. I also present recent empirical evidence and theoretical perspectives bearing on the importance of measuring the speed of making magnitude comparisons. Finally, I discuss how differing conceptions of number sense inform the issue of whether and to what extent it may be teachable.

Gersten, Jordan, and Flojo (in this issue) judiciously review and appraise relevant empirical evidence and theoretical perspectives pertaining to issues of both the early identification of mathematical disabilities (MD) and their remediation. Among other topics they cover in their incisive treatment of several pertinent literatures, Gersten et al. examine the construct of *number sense*. They begin by defining it conceptually and describing a strong research tradition and theoretical foundation for investigating its early development. Then, building on previous efforts to operationalize this construct, they present new evidence indicating that measures derived from this approach appear to be predictive of MD in kindergartners. Gersten et al. conclude this section by discussing past research on both formal and informal instructional interventions for improving number sense. Both their thoughtful review of this topic and the interesting original data they provide represent an important first step toward the design of valid screening measures for identifying chil-

dren at risk for developing MD. Notwithstanding Gersten et al.'s valuable contributions in this regard, several critical issues concerning the construct of number sense remain unresolved and, therefore, merit a more detailed examination. Such an analysis seems timely, given that half a century has passed since the concept of number sense was first delineated (Dantzig, 1954).

What Are the Components of Number Sense?

Gersten et al. (in this issue) point out that no two researchers define number sense in exactly the same way. What makes this situation even more problematic, however, is that cognitive scientists and math educators define the concept of number sense in very different ways. After perusing the relevant literature in the domains of mathematical cognition, cognitive development, and mathematics education, I compiled a list of presumed features of number sense, shown in Figure 1. (These components were drawn from

the following reports: Case & Sowder, 1990; Dantzig, 1954; Dehaene, 1997, 2001a; Fennell & Landis, 1994; Gersten & Chard, 1999; Howden, 1989; Kalchman, Moss, & Case, 2001; Markovits & Sowder, 1994; McIntosh, Reys, & Reys, 1992, 1997; McIntosh, Reys, Reys, Bana, & Farrell, 1997; Menon, 2004; National Council of Teachers of Mathematics [NCTM], 2000; Number Sense Research Group, 1995; Okamoto & Case, 1996a; B. J. Reys, 1994; R. Reys et al., 1999; Sowder, 1992; Verschaffel & De Corte, 1996; Yang, Hsu, & Huang, in press; Zanzali & Ghazali, 1999).

Inspection of Figure 1 reveals that number sense reputedly constitutes an awareness, intuition, recognition, knowledge, skill, ability, desire, feel, expectation, process, conceptual structure, or mental number line. Possessing number sense ostensibly permits one to achieve everything from understanding the meaning of numbers to developing strategies for solving complex math problems; from making simple magnitude comparisons to inventing procedures for conducting numerical operations; and from recogniz-

1. A faculty permitting the recognition that something has changed in a small collection when, without direct knowledge, an object has been removed or added to the collection (Dantzig, 1954).
2. Elementary abilities or intuitions about numbers and arithmetic.
3. Ability to approximate or estimate.
4. Ability to make numerical magnitude comparisons.
5. Ability to decompose numbers naturally.
6. Ability to develop useful strategies to solve complex problems.
7. Ability to use the relationships among arithmetic operations to understand the base-10 number system.
8. Ability to use numbers and quantitative methods to communicate, process, and interpret information.
9. Awareness of various levels of accuracy and sensitivity for the reasonableness of calculations.
10. A desire to make sense of numerical situations by looking for links between new information and previously acquired knowledge.
11. Possessing knowledge of the effects of operations on numbers.
12. Possessing fluency and flexibility with numbers.
13. Can understand number meanings.
14. Can understand multiple relationships among numbers.
15. Can recognize benchmark numbers and number patterns.
16. Can recognize gross numerical errors.
17. Can understand and use equivalent forms and representations of numbers as well as equivalent expressions.
18. Can understand numbers as referents to measure things in the real world.
19. Can move seamlessly between the real world of quantities and the mathematical world of numbers and numerical expressions.
20. Can invent procedures for conducting numerical operations.
21. Can represent the same number in multiple ways depending on the context and purpose of the representation.
22. Can think or talk in a sensible way about the general properties of a numerical problem or expression—without doing any precise computation.
23. Engenders an expectation that numbers are useful and that mathematics has a certain regularity.
24. A non-algorithmic feel for numbers.
25. A well-organized conceptual network that enables a person to relate number and operation.
26. A conceptual structure that relies on many links among mathematical relationships, mathematical principles, and mathematical procedures.
27. A mental number line on which analog representations of numerical quantities can be manipulated.
28. A nonverbal, evolutionarily ancient, innate capacity to process approximate numerosities.
29. A skill or kind of knowledge about numbers rather than an intrinsic process.
30. A process that develops and matures with experience and knowledge.

FIGURE 1. Alleged components of number sense.

ing gross numerical errors to using quantitative methods for communicating, processing, and interpreting information. With respect to its origins, some consider number sense to be part of our genetic endowment, whereas

others regard it as an acquired skill set that develops with experience.

The major disparity here is between what might be considered a “lower order” characterization of number sense as a biologically based “per-

ceptual” sense of quantity and a “higher order” depiction as an acquired “conceptual sense-making” of mathematics. The former view limits the features of number sense to elementary intuitions about quantity, including the rapid and accurate perception of small numerosities and the ability to compare numerical magnitudes, to count, and to comprehend simple arithmetic operations (Dehaene, 1997, 2001a; Geary, 1995). Although these foundational components are incorporated in the higher order perspective as well, number sense from this standpoint is considered to be much more complex and multifaceted in nature; it comprises a deep understanding of mathematical principles and relationships, a high degree of fluency and flexibility with operations and procedures, a recognition of and appreciation for the consistency and regularity of mathematics, and a mature facility in working with numerical expressions—all of which develop as a byproduct of learning through a wide array of mathematics education activities (Greeno, 1991; Verschaffel & De Corte, 1996; see Note 1).

Stanislas Dehaene (2001b), a cognitive neuroscientist and author of *The Number Sense* (Dehaene, 1997), contended that if only for the sake of economy, a single number sense (i.e., analog representation of quantity) should be hypothesized, rather than a patchwork of representations and abilities. However, he also suggested that this core representation becomes connected to other cognitive systems as a consequence of both development and education. “What is at stake is to what extent the initial quantity system is altered through these interactions; and to what extent the adult concept of number is based solely on the quantity system as opposed to an integration of multiple senses of numbers” (Dehaene, 2001b, p. 89).

Although Dehaene’s position seems eminently reasonable, at least to the present author, it is unlikely that math educators would embrace this view, given that the more expansive

conception of number sense they espouse is already entrenched in various forms, for example (a) as one of the five content standards of the National Council of Teachers of Mathematics *Principles and Standards for Mathematics* (2000); (b) in contemporary mathematics textbooks; and (c) as a distinct set of test items included in the mathematics portions of the National Assessment of Educational Progress (NAEP), the Trends in International Mathematics and Science Study (TIMSS), and the Program for International Student Assessment (PISA). Nonetheless, the present analysis can, I hope, inform the effort recommended by Gersten et al. (in this issue) to devise “more refined, better operationalized definitions of number sense” to make further progress in developing valid screening measures. That being said, one must bear in mind that the utility of operational definitions will in part be determined by the clarity and soundness of the theoretical conceptualizations from which they are derived. Indeed, Gersten et al. have proceeded in just such a manner by basing their screening measures on the models of both Geary (1993, 2004) and Case and colleagues (Kalchman et al., 2001; Okamoto & Case, 1996; see Note 2).

Why Are Timed Tests of Number Sense Important?

In discussing critical next steps for developing screening measures of number sense in general and numerical magnitude comparisons in particular, Gersten et al. (in this issue) suggest that future research be aimed at resolving whether measures administered to kindergartners and first graders should be timed or untimed. This recommendation was based on the reputed lack of evidence to indicate that the speed of making magnitude comparisons is an important variable to measure, at least with respect to its validity for predicting mathematical proficiency over the short term.

Interestingly, however, recent evidence from studies of school-age chil-

dren with MD suggests that it may be crucial to measure the speed of executing such quantity discriminations, as this variable can reveal subtle yet important differences in numerical information processing that may not be tapped by assessing accuracy alone. For example, by measuring response times on individual magnitude comparison trials composed of single-digit pairs, Landerl, Bevan, and Butterworth (2004) found that 8- and 9-year-old children with dyscalculia were significantly slower than controls for correct responses, despite a lack of difference in error rates (but the groups did not differ in their speed of making physical size comparisons). Similarly, Passolunghi and Siegel (2004) discovered that whereas fifth-grade children with MD were no less accurate in comparing the magnitudes of one- to four-digit numbers than their typically achieving controls, they were significantly slower overall in carrying out these comparisons for a set of 16 number pairs.

As the vast majority of item pairs for the Passolunghi and Siegel (2004) study were composed of two- to four-digit numbers, it is likely that further investigation of numerical comparisons for multidigit numbers may prove useful. Indeed, as Geary and Hoard (2002) have pointed out, although a good deal is known about the magnitude comparison (and other numerical comprehension) skills of children with MD for single-digit numbers, relatively little is known about such abilities with respect to more complex numerals. Some recent research examining the speed with which typically achieving children of various ages make magnitude comparisons with two-digit numbers has yielded an intriguing hypothesis about a source of deficits in the multidigit number judgments of children with MD.

Before discussing this study, it is necessary to briefly review the results of prior, related work with adults. First, when making magnitude comparisons with two-digit numbers, adults exhibit a so-called *distance effect* (Dehaene, Dupoux, & Mehler, 1990); that

is, the greater the numerical difference between the two-digit numbers being compared, the shorter the time required to judge which number is larger (see Notes 3 and 4). For example, adults respond more quickly when judging whether 57 is larger than 42 than when deciding whether 61 is larger than 59. This result is counterintuitive, as the size of the unit digits is irrelevant when the decade digits differ in magnitude. Furthermore, the finding of a distance effect was taken to indicate that the decade and unit digits are mapped holistically on the mental number line rather than separately. However, Nuerk, Weger, and Willmes (2001) subsequently showed that *incompatible* number pairs, in which the unit and decade digits lead to different “decisions” (e.g., 47 and 62) were responded to more slowly than compatible number pairs (e.g., 42 and 57). This so-called *unit-decade compatibility effect* suggests that at least in adults, two-digit numbers are processed separately and in parallel rather than holistically.

By examining response times for similar types of two-digit number comparisons in second to fifth graders, Nuerk, Kaufmann, Zoppoth, and Willmes (2004) recently demonstrated that independent processing of the decade and unit digits begins around the second grade. Moreover, their results suggested that this strategy of decomposing the decade and unit digits develops from a sequential (left to right) processing mode to a more parallel processing mode. Nuerk et al. contended that what develops with age, then, is not the kind or number of representations, but the way in which children access and integrate them.

Of particular importance with respect to the present analysis are the implications of these findings for children who experience MD. Namely, Nuerk et al. hypothesized that the development of rapid, quasi-parallel digit integration when working with multidigit Arabic numbers may be deficient in children with MD. As they pointed out, if digit integration develops more slowly or is still predominantly se-

quential in such children, then the calculation of multidigit numbers would demand considerably more working memory and attentional resources than if the digits are integrated in a rapid, quasi-parallel fashion.

It is interesting to note that both of the two-digit magnitude comparison items from the *Number Knowledge Test* (Okamoto & Case, 1996) shown in Table 1 of Gersten et al. (in this issue; i.e., "Which is bigger: 69 or 71?" and "Which is smaller: 27 or 32?") constitute incompatible number pairs. Based on the findings of Nuerk et al. (2004), it is possible that using only incompatible pairs may inadvertently yield evidence of even greater processing difficulties for children with MD than if compatible pairs (e.g., 65 and 78) were also included, at least from the second grade on. Unfortunately, we do not as yet know whether a unit-decade compatibility effect would emerge in kindergartners or first graders, even with respect to error rates. Consequently, further study of this issue is likely to prove informative both for theory and for early identification.

Can Number Sense Be Taught?

Gersten et al. (in this issue) claim that if number sense is viewed as a skill or a kind of knowledge rather than an "intrinsic" process (Robinson et al., 2002), it should be teachable. However, as some theorists have claimed that number sense is rooted in our biological makeup, what are the implications of this perspective for "teaching" it or at least fostering its development? Contrary to a strict nativist position, most theorists who adhere to the view that number sense has a long evolutionary history and a specialized cerebral substrate do not judge that it thereby constitutes a fixed or immutable entity. Rather, the emergence of rudimentary components of number sense in young children is thought to occur "spontaneously without much explicit instruction" (Dehaene, 1997, p. 245), at least

under nondisadvantaged rearing conditions.

Furthermore, according to Geary (1995), the neurocognitive systems supporting these elementary numerical abilities include what has been referred to as *skeletal principles* (Gelman, 1990; Gelman & Meck, 1992), because they provide just the foundational structure for the acquisition of these abilities. Concomitantly, engaging in numerical kinds of games and activities is thought to "flesh out" these principles (Geary, 1995). Consistent with this perspective, the effective use of board games with children with low socioeconomic status (Griffin, Case, & Siegler, 1994) mentioned by Gersten et al. (in this issue) aptly leads them to the conclusion that both formal and informal instruction can enhance number sense development prior to entering school. The interested reader should consult Siegler and Booth (2004) for a thoughtful delineation of the multiple factors inherent in such games that are ideal for enhancing the construction of a linear mental representation of numerical magnitude, as well as for a variety of practical suggestions regarding other instructional strategies that can be used at home or at school.

Gersten et al. (in this issue) also suggest that one goal of early intervention is to enhance the ability of children to use a mental number line. That being said, the most judicious approach to selecting an intervention would be to base it on one's model of the mental number line, as Gersten et al. have done with respect to Case's central conceptual structures model. How do these various models differ, and what kinds of instructional interventions can be derived from them? Given space limitations, only a brief account of these issues can be given here. Basically, there is general agreement among such theorists that at its most nascent level, numerical information is probably manipulated in an analog format in which numerosities are represented as distributions of activation on the mental number line (Dehaene, 2001a). However, the precise nature of this repre-

sentation is as yet unresolved (Siegler & Booth, 2004). Nonetheless, there is a growing body of evidence that this system putatively evolved to handle only approximate numerical judgments, not exact ones (Dehaene, 1997, 2001a; Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999; Pica, Lemer, Izard, & Dehaene, 2004).

As Geary (in press) pointed out, "the learning of specific quantities beyond four and mapping number words onto the representations of these quantities is a difficult task, because the analog magnitude system that must be adapted for this purpose functions to represent general amounts, not specific quantities (Gallistel & Gelman, 1992)." The difficulties associated with this mapping process can be especially challenging for children with MD who also experience reading difficulties. For example, Hanich, Jordan, Kaplan, and Dick (2001) found that children with comorbid MD and reading difficulties performed lower on a measure of exact arithmetic than children with MD only, although these groups did not differ on a test of approximate arithmetic. As Gersten et al. (in this issue) correctly note, different interventions may be required for ameliorating the difficulties experienced in these two types of MD.

What are the pedagogical implications of viewing number sense as a much more complex and multifaceted construct than "simply" possessing elementary intuitions about quantity? Among other recommendations emanating from this perspective, it has been argued that number sense "cannot be compartmentalized into special textbook chapters or instructional units" (Verschaffel & De Corte, 1996, p. 109) and that its development does not result from a selected subset of activities designed specifically for this purpose (Greeno, 1991). Similarly, B. J. Reys (1994) contended that number sense constitutes "a way of thinking that should permeate all aspects of mathematics teaching and learning" (p. 114). Finally, Greeno (1991) suggested that "it may be more fruitful to view num-

ber sense as a by-product of other learning than as a goal of direct instruction" (p. 173). It should be noted that these strategies, along with other "constructivist" approaches, have been considered by some to be only weakly related to the theoretical conceptions and empirical findings that have emerged from the cognitive sciences (Geary, 1995).

Conclusion

The objective of the present analysis was to shed some light on the nature, origins, and pedagogical implications of differing conceptions of number sense. Although various definitional and theoretical issues pertaining to this construct remain unresolved, there appear to be several promising, model-based directions for both the early identification of potential mathematical difficulties and the design of instructional interventions. By summarizing the major issues in this domain, framing the critical questions, and suggesting some next steps in developing valid screening measures and effective intervention programs, Gersten et al. (in this issue) have performed a valuable service for the field. I hope the preceding examination of number sense will contribute to this important endeavor.

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NOTES

1. Consistent with this broad conception, several other types of mathematical "senses" have also been proffered: an operation sense, a graphic sense, a spatial sense, a symbol sense, and a structure sense (Arcavi, 1994;

Linchevski & Livneh, 1999; Picciotto & Wah, 1993; Slavit, 1998).

2. It should be pointed out that the comparatively broad-based, higher order conception of number sense espoused by mathematics educators has yielded a surfeit of numerical and mathematical tasks, problems, and test items that operationalize number sense from pre-K all the way through high school (see Number Test Item Bank at <http://www.nzmaths.co.nz/numeracy/Other%20material/Number%20Sense%20Items.pdf>).
3. The distance effect for magnitude comparisons with single digits was first reported almost 40 years ago (Moyer & Landauer, 1967). Moreover, there are several lines of evidence indicating the importance of examining this speed-based effect (although it also shows up in error rates): (a) the distance effect has been found to occur in children as young as 5 years of age (Duncan & McFarland, 1980; Sekular & Mierkiewicz, 1977; Temple & Posner, 1998); (b) this effect is one of the key markers of the mental number line; (c) the neuroanatomical substrate of the comparison process that yields this effect appears to be the same in young children as in adults (Temple & Posner, 1998); and (d) the slope of the response time function representing the distance effect systematically declines in children as they get older (Sekular & Mierkiewicz, 1977). One explanation for the latter outcome is that the analog quantity representations on the mental number line may become less "noisy" with age and experience.
4. The only study to date that has assessed speed of magnitude comparisons at different distance levels in children with MD is the Landerl et al. (2004) experiment with 8- and 9-year olds described earlier. Neither the MD-only nor the MD + RD group exhibited a distance effect; however, as neither the RD-only group nor the typically achieving controls did either, the possibility of differences in the distance functions between children with and without MD remains to be determined. Some evidence regarding this issue can be gleaned from the only two other studies that have examined the effects of numerical distance on performance in children with MD, despite the fact that only accuracy levels were measured (Geary, Hamson, & Hoard, 2000; Geary, Hoard, & Hamson, 1999). In the first study (Geary et al., 1999), no distance effect emerged in first grade for either the MD-only, RD-only, or typically achieving children, with performance being near ceiling for the latter two groups. However, one third of the MD + RD group chil-

dren could not correctly judge the larger number for either small-value (e.g., 2 and 3) or large-value (e.g., 8 and 9) pairs at a distance level of 1. Assessing these same children again in second grade, Geary et al. (2000) found that most of the MD + RD children could determine that 3 is more than 2, although they still did not know that 9 is larger than 8 (a finding known as the size effect). These findings, limited though they may be, suggest to the present author that the analog representations of arabic numerals on the mental number line of MD + RD group children may be "noisier" than for typically achieving children. Clearly, further investigation of the distance effect for single digits in children with MD is needed in order to test this hypothesis.

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Test of Early Mathematics Ability—Third Edition

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The TEMA-3 measures the mathematics performance of children between the ages of 3-0 and 8-11 and is also useful with older children who have learning problems in mathematics. Used as a norm-referenced measure or a diagnostic instrument, the test can be used to measure progress, evaluate programs, screen for readiness, discover the bases for poor school performance in mathematics, identify gifted students, and guide instruction and remediation. The test measures informal and formal (school-taught) concepts and skills in the following domains: numbering skills, number-comparison facility, numeral literacy, mastery of number facts, calculation skills, and understanding of concepts. It has two parallel forms, each containing 72 items.

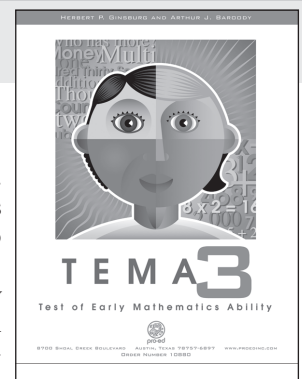
The characteristics of the sample of 1,219 children approximate those in the 2001 U.S. Census. Internal consistency reliabilities are all above .92; immediate and delayed alternative form reliabilities are in the .80s and .90s.

Also provided is a book of remedial techniques (Assessment Probes and Instructional Activities) for improving skills in the areas assessed by the test.

Several important improvements were made in the TEMA-3.

a) A linear equating procedure is used to adjust scores on the two tests so they may be used interchangeably. b) Bias studies show the absence of bias based on gender and ethnicity. c) The pictures of animals and money in the Picture Book are now in color to make them more appealing and more realistic in appearance.

COMPLETE TEMA-3 KIT INCLUDES: Examiner's Manual, Picture Book Form A, Picture Book Form B, 25 Examiner Record Booklets Form A, 25 Examiner Record Booklets Form B, 25 Worksheets Form A, 25 Worksheets Form B, Assessment Probes, 5" x 8" cards, 20 blocks, 20 tokens, and a mesh bag, all in a sturdy storage box. (2003)



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